Hypothesis Testing : 1-Sample t-Test

Consider the following example:

Research Question: *Is the mean nose length of Tasmanian spotted-aardvarks greater than 9.2 inches?*

Null Hypothesis wording: *The mean nose length of Tasmanian spotted-aardvarks is less than or equal to 9.2 inches.*

Null Hypothesis symbolically: H_0: $\mu \leq 9.2$

Alternative Hypothesis wording: *The mean nose length of Tasmaian spotted-aardvarks is greater than 9.2 inches.*

Alternative Hypothesis symbolically: H_0: $\mu > 9.2$

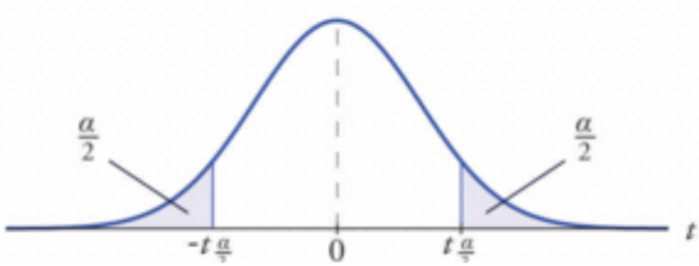
This kind of hypothesis test will involve the use of a 1-sample t-test that determines the statistical significance of the relationship between the sample mean and the proposed population parameter.

Note: For samples greater than or equal to 30, you can assume a normal distribution and use z-score calculations for your hypothesis tests. However, since the t-distribution approximates the normal curve for sample sizes over 30, we will only explore the application of t-tests below.

<u>One-Sample T-test</u>

When you compare the mean of your sample to some hypothesized population mean, the null hypothesis and alternative hypothesis will follow one of the following formats:

2-tailed test:

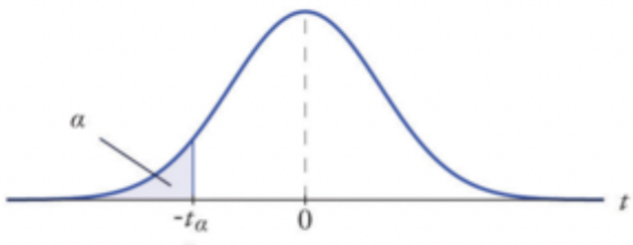


Ho: μ = somevalue

Ha: μ ≠ somevalue

For the 2-tailed test, you have two rejection regions (if $\alpha = .05$, then it will be .025 each).

1-tailed test: left

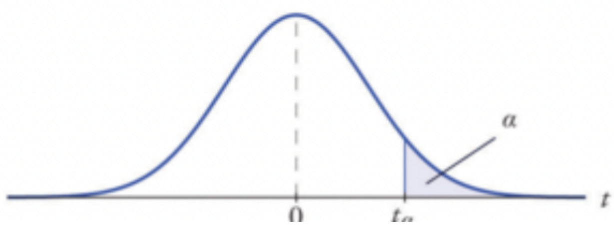


Ho: μ ≤ somevalue

Ha: μ > somevalue;

Rejection region to the left of the critical value.

1-tailed test: right



Ho: μ≥somevalue

Ha: μ<somevalue

Rejection region to the right of the critical value.

Let's take a look at a couple of examples of a 1-sample t-test.

Example 1: The nap times of a random sample of 25 Nifflers was observed with a mean of 22.1 minutes and a standard deviation of 5.3 minutes. Is the mean nap time of Nifflers less than 25.4 minutes? Use a level of significance of $\alpha = .05$.

H_o: $\mu \geq 25.4$

H_a: $\mu < 25.4$

n = 25; df = 25 - 1 = 24

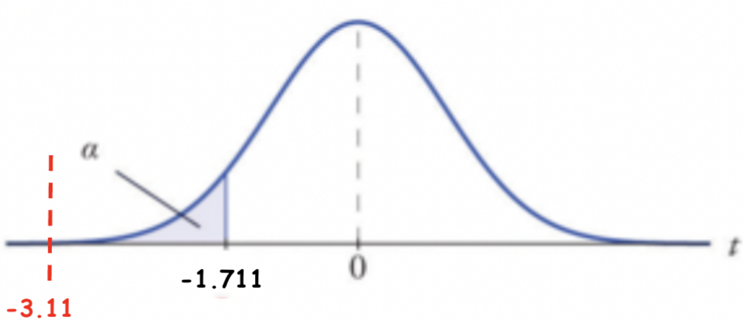
$\alpha = .05$

t-critical = -1.711 found using https://goodcalculators.com/student-t-value-calculator/

This will be a left-tailed test.

Our t-value will be:

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{22.1-25.4}{\frac{5.3}{\sqrt{25}}} = -3.11$$



Since our t calculated is to the left of our t-critical, we will reject the null hypothesis suggesting there is sufficient evidence to support the alternative hypothesis.

Inference Drawn: *Based upon the data collected, it appears that the mean nap time for Nifflers is less than 25.4 minutes.*

Example 2: A random sample of 20 coding camp students was surveyed and the mean cost to attend these camps was determined to be $15,560 with a standard deviation of $3,500. At a significance level of .05, determine if the mean cost to attend coding camps is greater than $13,252.

H_o: $\mu \leq 13,252$

H_a: $\mu > 13,252$

n = 20; df = 20 - 1 = 19

$\alpha = .05$

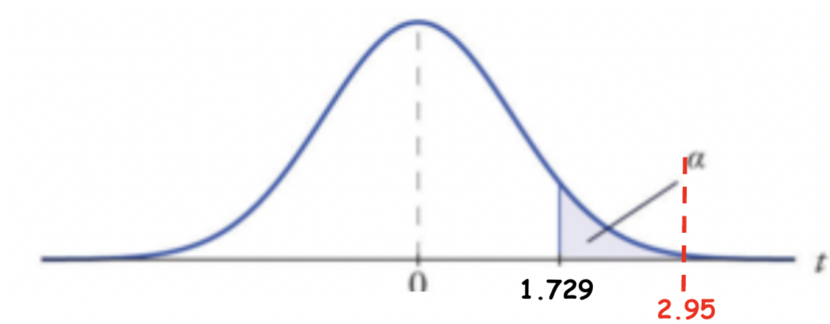
t-critical = 1.729 found using https://goodcalculators.com/student-t-value-calculator/

This will be a right-tailed test.

Our t-value will be:

$$t = \frac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{15560-13252}{\frac{3500}{\sqrt{20}}} = 2.95$$

Since our t calculated is to the right of our t-critical, we will reject the null hypothesis suggesting there is sufficient evidence to support the alternative hypothesis.



Inference Drawn: *Based upon the data collected, it appears that the mean costs to attend a coding camp is greater than $13,252.*

Example 3: A random sample of 16 homes in Statsville, USA resulted in a mean daily water usage of 119 gallons with a standard deviation of 5.3 gallons. At a .05 level of significance, determine if the mean usage of water in the community is different that 123 gallons.

H_o: μ = 123

H_a: $\mu \neq 123$

n = 16; df = 16 - 1 = 17

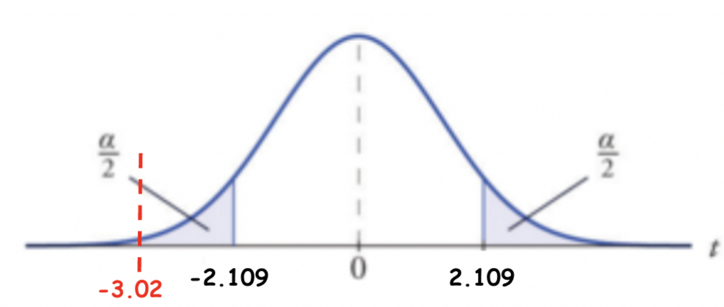
$\alpha = .05$

t-critical = $\pm\ 2.109$ found using https://goodcalculators.com/student-t-value-calculator/

This will be a two-tailed test.

Our t-value will be:

$$t = \frac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{119-123}{\frac{5.3}{\sqrt{16}}} = -\ 3.02$$



Our t calculated value falls in the left rejection region. So, we reject the null hypothesis here because there is sufficient evidence to support the alternative hypothesis.

Inference Drawn: *Based upon the data collected, it appears that the mean daily water usage in Statsville, USA does not equal 123 gallons.*

Yes, those were all examples of rejecting the null hypothesis.

Let's see a few examples where we fail to reject the null hypothesis:

Example 4: A random sample of 15 homes were surveyed and it was determined that the mean monthly cost for XYZ streaming services was $18.72 with a standard deviation $3.64. Determine if the mean monthly cost (for all users) of XYZ streaming services is different than $17.63 ($\alpha = .05$).

H_o: μ = $17.63

H_a: $\mu \neq \$17.63$

n = 15; df = 15 - 1 = 14

$\alpha = .05$

t-critical = $\pm\ 2.145$ found using https://goodcalculators.com/student-t-value-calculator/

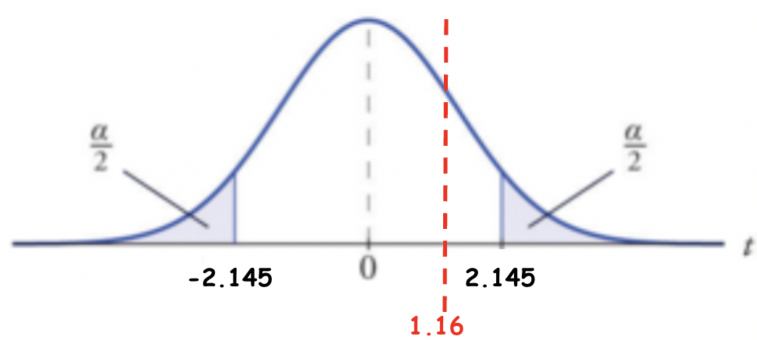
This will be a two-tailed test.

Our t-value will be:

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{18.72 - 17.63}{\frac{3.64}{\sqrt{15}}} = 1.16$$

Since our t calculated value is between our t-critical values, we will fail to reject the null hypothesis.

This means that there is insufficient evidence to support the alternative hypothesis.



Inference Drawn: *Based upon the data collected, it appears there is not sufficient evidence to suggest that the mean monthly cost for XYZ streaming services is different than $17.63.*

Example 5: From a random sample of 25 peanut farms in Virginia, it was determined that the mean number of peanuts per acre was 3120 with a standard deviation of 578. Is the mean number of peanuts per acre for all farms in Virginia greater than 3000 ($\alpha = .05$)?

H_o: $\mu \leq 3000$

H_a: $\mu > 3000$

n = 25; df = 25 - 1 = 24

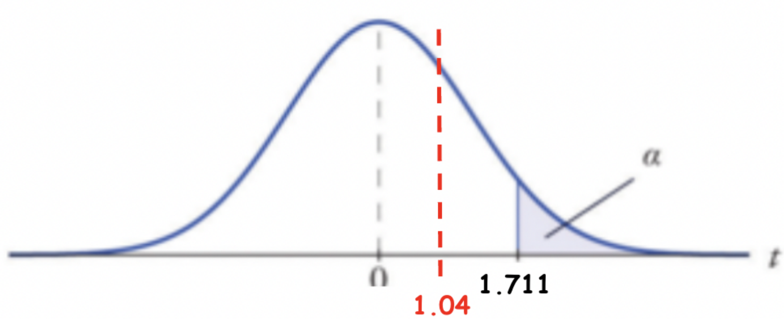
$\alpha = .05$

t-critical = 1.711 found using https://goodcalculators.com/student-t-value-calculator/

This will be a right-tailed test.

Our t-value will be:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3120 - 3000}{\frac{578}{\sqrt{25}}} = 1.04$$



Our t calculated value is to the left of our t-critical (outside the rejection region), so we fail to reject the null hypothesis.

Inference Drawn: *Based upon the data collected, it appears there is not sufficient evidence to suggest that the mean* number of peanuts per acre for all farms in Virginia is greater than 3000.

There are many ways to conduct a 1-sample t-test from raw sample data. You will be provided an Excel worksheet in the class from these calculations. You can also use one of the mean freely available online 1-sample t-test calculators such as the following:

https://www.socscistatistics.com/tests/tsinglesample/default.aspx

The Level of Significance

When a researcher conducts a study, the level of statistical significance is typically determined before the hypothesis test is performed. The level of significance, α, the probability of rejecting the null hypothesis when it is true. For example, a significance level of .05 indicates a 5% risk of rejecting a correct null hypothesis. While it is possible to select any value for the level of significance, researchers will select the lowest percentage that they feel will be acceptable for their research. The most common level of significance is α=.05 which also implies a 95% confidence level.

The P-value

The results of a hypothesis test will always report a p-value. The p-value represents the probability associated with the test statistics assuming that the null hypothesis is valid. In practice, the p-value is used as a decision value when compared to the selected level of significance α to determine if the null hypothesis should be rejected. A small p-value will mean that there is strong evidence to support the alternative hypothesis.

If p−value < α, then *reject the null hypothesis*. This means that there is sufficient evidence to support the alternative hypothesis.

If p−value ≥ α, then *fail to reject the null hypothesis*. This result means that there is not sufficient evidence to support the alternative hypothesis.

Let's revisit some of our previous examples of hypothesis testing, but this time we will determine the p-value of the test and compare it to the level of significance to determine the fate of the null hypothesis.

Example 1: The nap times of a random sample of 25 Nifflers was observed with a mean of 22.1 minutes and a standard deviation of 5.3 minutes. Is the mean nap time of Nifflers less than 25.4 minutes? Use a level of significance of $\alpha = .05$.

H_o: $\mu \geq 25.4$ H_a: $\mu < 25.4$

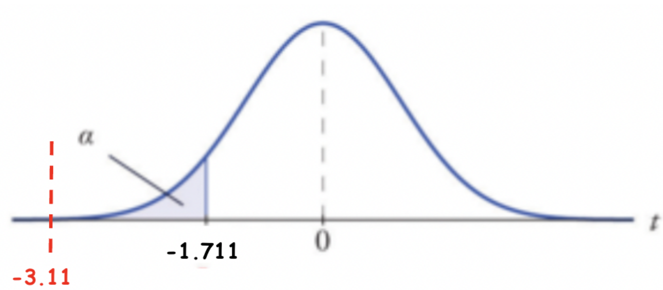
n = 25; df = 25 - 1 = 24 $\alpha = .05$

t-critical = -1.711 found using https://goodcalculators.com/student-t-value-calculator/

Use https://www.socscistatistics.com/pvalues/tdistribution.aspx to find the p-value from the t-score and degrees of freedom.

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{22.1-25.4}{\frac{5.3}{\sqrt{25}}} = -3.11$

p-value: $p(t \geq -3.11) = .002386$



Results: Since the p-value < α (.002386 < .05), we will *reject the null hypothesis*.

Inference Drawn: *Based upon the data collected, it appears that the mean nap time for Nifflers is less than 25.4 minutes.*

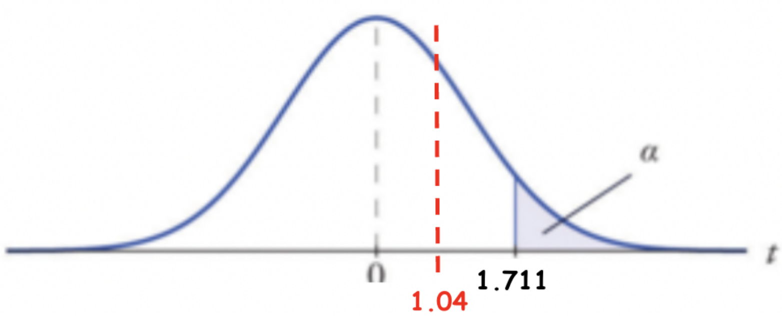
Example 5: From a random sample of 25 peanut farms in Virginia, it was determined that the mean number of peanuts per acre was 3120 with a standard deviation of 578. Is the mean number of peanuts per acre for all farms in Virginia greater than 3000 ($\alpha = .05$)?

H_o: $\mu \leq 3000$ H_a: $\mu > 3000$

n = 25; df = 25 - 1 = 24 $\alpha = .05$

t-critical = 1.711 found using https://goodcalculators.com/student-t-value-calculator/

Our t-value will be:



$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{3120-3000}{\frac{578}{\sqrt{25}}} = 1.04$$

p-value: $p(t \leq 1.04) = 0.1543$

Results: Since the p-value > α (.1543 > .05), we will *fail to reject the null hypothesis*.

Inference Drawn: *Based upon the data collected, it appears there is not sufficient evidence to suggest that the mean* number of peanuts per acre for all farms in Virginia is greater than 3000.