Kruskal-Wallis Test

The Kruskal-Wallis test is a non-parametric statistical test used to determine whether there are statistically significant differences between the median rank differences of three or more independent groups. It is an extension of the Mann-Whitney U test which compares two groups. In a sense, the Kruskal-Wallis test is essentially a one-way ANOVA test for ranks that does not require normality.

Null Hypothesis: There is no difference between the groups.
Alternative Hypothesis: There is a difference between the groups.
Alpha = .05

The test statistic for the Kruskal-Wallis test, denoted as H, is calculated using the formula:

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$$

where:

- N is the total number of observations across all groups.
- k is the number of groups.
- R_i is the sum of ranks for the *i*-th group.
- n_i is the number of observations in the *i*-th group.

Steps to Calculate H:
1. Rank all the observations from all groups together, assigning the average rank in the case of ties.
2. Sum the ranks for each group R_i.
3. Use the formula above to calculate the test statistic H.

The test statistic H approximately follows a chi-square distribution with k - 1 degrees of freedom, where k is the number of groups. To determine if the null hypothesis can be rejected, the H test value is compared to an H critical value obtained from a chi-square table by cross reference the degrees of freedom (df) with the level of significance (α): Here are a couple of different chi-square critical values tables that can be referenced:

https://courses.wccnet.edu/~palay/math160r/chisqtable.htm
https://math.arizona.edu/~jwatkins/chi-square-table.pdf.

Interpreting the Results:

H test < H critical - Fail to reject the null hypothesis.
H test > H critical - Reject the null hypothesis.

In most statistical software, the Kruskal-Wallis test returns a p-value, which is used to assess whether the observed differences in rank sums are due to chance. If the p-value is below a chosen significance level (e.g., 0.05), the null hypothesis is rejected, indicating at least one group median is significantly different.

Here are some online calculators:

https://www.socscistatistics.com/tests/kruskal/default.aspx#google_vignette

https://www.statskingdom.com/kruskal-wallis-calculator.html

Example: Comparing Dog Breeds

A veterinary researcher is interested in comparing three samples of different dog breeds to see if there is a statistical difference in their weights. The data can not be assumed to be normally distributed, so the researcher decides to apply the Kruskal-Wallis Test to test the ranks rather than the means. The data to be explored is shown below:

Breed A	Breed B	Breed C
65	80	70
72	78	65
58	85	72
60	75	68
70	82	75
62	77	63
68	83	77
55	70	60
67	88	73
75	79	67

To conduct our analysis, follow these steps:

Step 1: Combine all data in one list

Step 2: Sort by variable (weight)

Step 3: Rank each data point taking into account all ties.

Step 4: Reorganize the data by samples.

The results are shown in the following table.

Breed A	Rank A	Breed B	Rank B	Breed C	Rank C
55	1	70	14	60	3.5
58	2	75	20	63	6
60	3.5	77	22.5	65	7.5
62	5	78	24	67	9.5
65	7.5	79	25	68	11.5
67	9.5	80	26	70	14
68	11.5	82	27	72	16.5
70	14	83	28	73	18
72	16.5	85	29	75	20
75	20	88	30	77	22.5

Step 5: Calculate the rank sums for each sample.

	Rank Sum
A	90.5
B	245.5
C	129

Step 6: Calculate the H test statistic.

N = 30, n = 10, k = 3, and our ranks sums are in the table above. Applying the H formula:

$$H = \frac{12}{30(30+1)}\left(\frac{90.5^2}{10} + \frac{245.5^2}{10} + \frac{129^2}{10}\right) - 3(30 + 1) = 16.81$$

Step 7: Determine the H critical value at $\alpha = .05$ and N = 30.

From the chi-squared table, H-critical = 5.991

Step 8: Results.

Since our H test value of 16.81 is greater than the H critical of 5.991, it is outside of the chi-square acceptance range of (0, 5.991) which means that we will reject the null hypothesis.

Here's how to perform this analysis in R-studio:

```
# Sample data
group <- factor(rep(1:3, each = 10))
value <- c(65, 72, 58, 60, 70, 62, 68, 55, 67, 75, 80, 78, 85, 75, 82, 77, 83, 70, 88, 79, 70, 65, 72, 68, 75, 63, 77, 60, 73, 67)

# Create a data frame
data <- data.frame(value, group)

# Perform Kruskal-Wallis test
kruskal.test(value ~ group, data = data)
```

This results in an H = 16.861, df = 2, p-value = 0.0002181. Since this p-value is less than .05, then the null is rejected.