1-LINEAR EQUATIONS

In this week's lecture, we are discussing the equation of a line (a.k.a. linear equations).

The basic form for a linear equation is as follows:
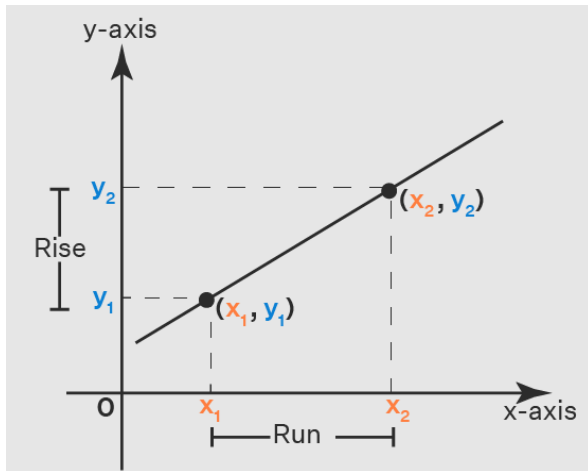
$$y = mx + b$$

Where m is the slope of the line and b is the y-intercept.

This is known as the **slope-intercept form** for the equation of a line.

If you are given any two points, (x_1, y_1) and (x_2, y_2), on the coordinate plane, you can use them to determine the equation of the line through those points.

The first step is to find the slope of the line using the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line determines the tilt of the line given the amount of change between the y values (the rise) divided by the amount of change between the x values (the run). That's why you'll often hear the slope described as rise over run.

$$m = \frac{rise}{run}$$

Once you've determined the slope, you can use the following **point-intercept form** for the equation of a line.

$$y - y_1 = m(x - x_1)$$

Where m is the slope just determined and (x_1, y_1) are one of the given points.

Example: Find the equation of the line through the points (-3, 2) and (6, -4).

First, we calculate the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4 - 2}{6 - (-3)} = \frac{-6}{9} = -\frac{2}{3}$$

Next, we plug this value for the slope and the values for the point (-3, 2), into the point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$
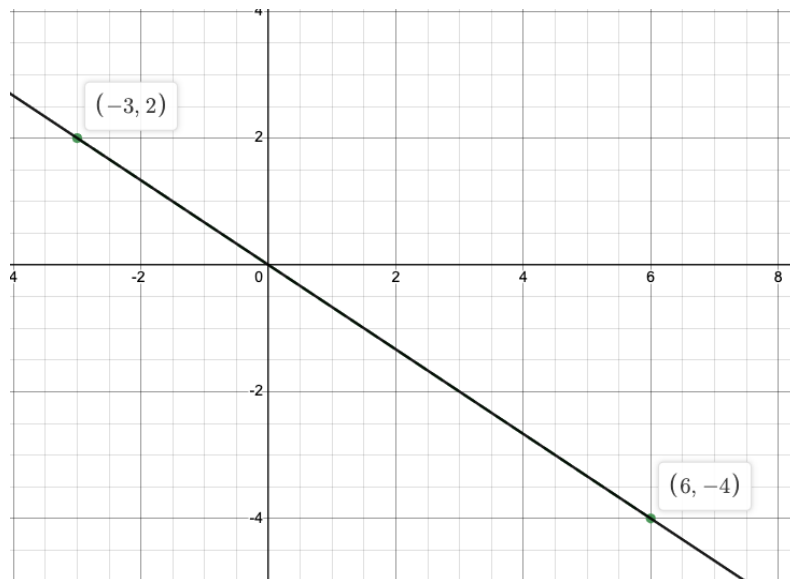
$$y - 2 = -\frac{2}{3}(x - (-3))$$

$$y - 2 = -\frac{2}{3}(x + 3)$$

$$y = -\frac{2}{3}x + (-\frac{2}{3})(3) + 2$$

$$y = -\frac{2}{3}x + (-2) + 2$$

$$y = -\frac{2}{3}x$$

The equation that we've just found is in the form $y = mx + b$ where the slope is $m = -\frac{2}{3}$ and the y-intercept is $b = 0$.   Here's how this equation would look when drawn on the coordinate plane using https://www.desmos.com/calculator.

This particular line crosses through the point (0, 0) also called the origin. As such the x-intercept for this line is zero as well.

Example: Find the equation of the line through the points (3, 6) and (-6, -4).

First, determine the slope of the line:

$$m = \frac{-4-6}{-6-3} = \frac{-10}{-9} = \frac{10}{9}$$

Next, using $m = \frac{10}{9}$ and the point (3, 6), employ the point-slope form to determine the equation of the line:

$$y - 6 = \frac{10}{9}(x - 3)$$

$$y - 6 = \frac{10}{9}x - \left(\frac{10}{9}\right)(3)$$
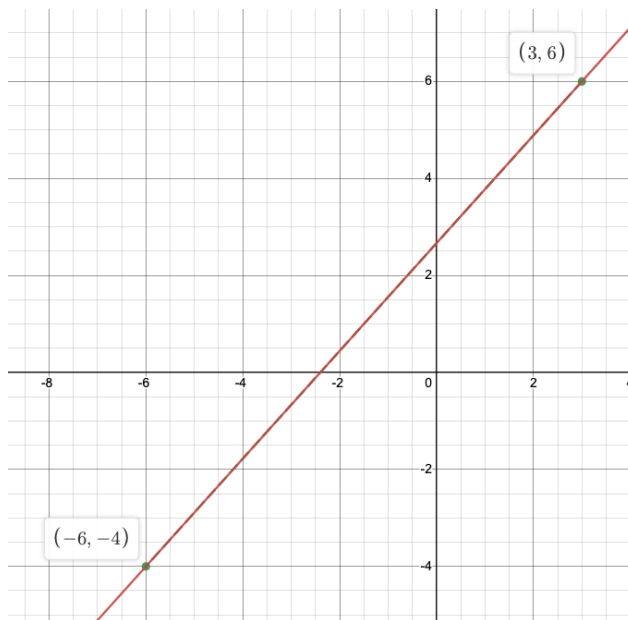
$$y - 6 = \frac{10}{9}x - \frac{10}{3}$$

$$y = \frac{10}{9}x - \frac{10}{3} + 6$$

$$y = \frac{10}{9}x - \frac{10}{3} + \frac{18}{3}$$

$$y = \frac{10}{9}x + \frac{8}{3}$$

The equation that we've just found is in the form $y = mx + b$ where the slope is $m = \frac{10}{9}$ and the y-intercept is $b = \frac{8}{3}$. Here's how this equation would look when drawn on the coordinate plane using https://www.desmos.com/calculator.

So now you know how to find the equation of the line between any two points. Let's move on to finding the intercepts for a linear equation.

Example: Find the x and y intercepts for the linear equation $y = 5x - 1$

The y-intercept is the point at which the line crosses the y-axis. As such, the x coordinate at the point will be zero. So, to determine the coordinates for the y-intercept, you will set x = 0 in your linear equation.

$$y = 5(0) - 1$$

$$y = 0 - 1$$

$$y =- 1$$

So, when x = 0, the y-intercept value is y = -1. That means the y-intercept point will be (0, -1).

The x-intercept is the point at which the line crosses the x-axis. As such, the y coordinate at the point will be zero. So, to determine the coordinates for the x-intercept, you will set y = 0 in your linear equation.
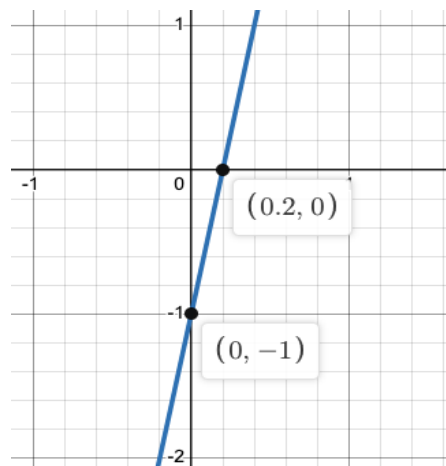
$$0 = 5x - 1$$

$$- 5x = - 1$$

$$x = \frac{-1}{-5} = \frac{1}{5}$$

So, when y = 0, the x-intercept value is $x = \frac{1}{5} = 0.2$. That means the x-intercept point will be (0.2, 0).

Here's the graph of $y = 5x - 1$ with intercepts labeled:

Note that this is an example of a line with a positive slope.

Example: Find the x and y intercepts for the linear equation $y = -\frac{2}{5}x - 4$

y-intercept:

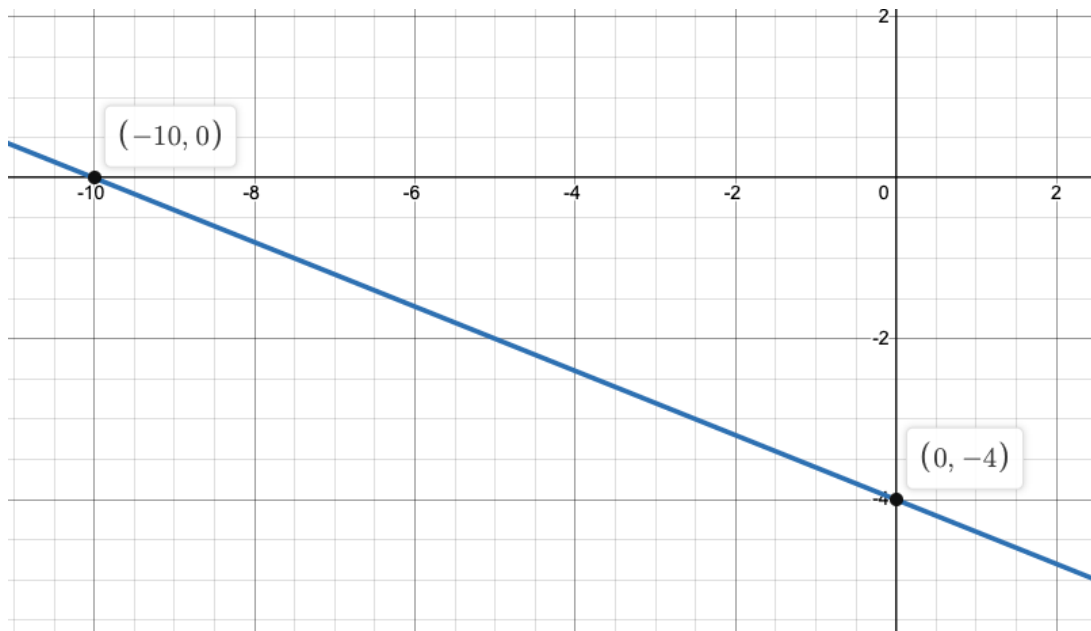$$y = -\frac{2}{5}(0) - 4$$

$$y = \quad - 4$$

x-intercept:

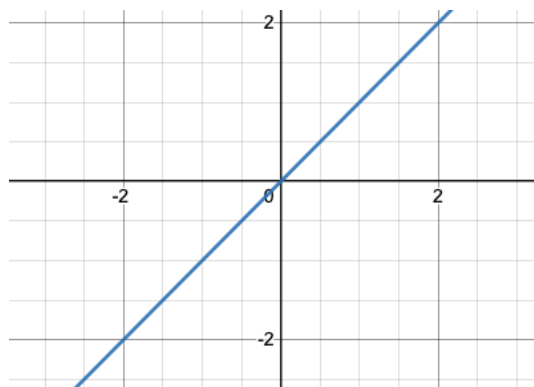$$0 = -\frac{2}{5}x - 4$$

$$\frac{2}{5}x = \quad - 4$$

$$x = (\frac{5}{2})(- 4) = (5)(- 2) = \quad - 10$$

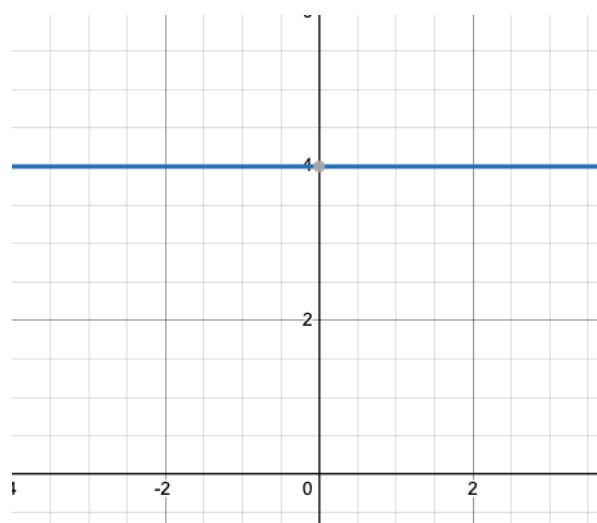Graph of $y = -\frac{2}{5}x - 4$ with intercept points labeled.

Note that this is an example of a line with a negative slope.

The graph of the equation $y = x$ is an example of a line with a slope of 1.

The graph of the equation y = 4 is an example of a line with a slope of zero.

Finally, we have the graph of the equation x = 3. The slope of this line is referred to as *undefined*.