

Hypothesis Testing: 2-Sample t-Test

Comparing Sample Means (Independent Means)

Let's say that you have the following research question in which the mean of one sample is compared to the mean of another sample.

Does the mean weight of Texas White-tailed deer in 2000 differ from the mean weight of Texas White-tailed deer in 2020?

Here's how you would format the null hypothesis:

H_0 : The mean weight of Texas White-tailed deer in 2000 does not differ from the mean weight of Texas White-tailed deer in 2020.

$$H_0: \mu_1 = \mu_2$$

Here's how you would format the alternative hypothesis:

H_a : The mean weight of Texas White-tailed deer in 2000 differs from the mean weight of Texas White-tailed deer in 2020.

$$H_a: \mu_1 \neq \mu_2$$

This type of question is generally known as a comparison of the means. The statistical test employed is a 2-sample t-test.

Two-Sample t-test

If you are comparing the means of two samples, then your default null is that there is a difference and the alternative will be that there is no difference. This test is a 2-tailed test by default.

$$H_0: \mu_1 = \mu_2$$

$$H_a: \mu_1 \neq \mu_2$$

To conduct this type of hypothesis test, we will avoid using the required formulas (as shown in the text) and rely upon software to compute the necessary test statistics and p-values.

An Excel worksheet will be provided for you to perform 2-sample t-test calculations. The following online calculator will perform a 2-sample t-test for two independent means. To use this test, you must copy your data (numbers only) into the slots provided and it will generate what you need.

<https://www.socscistatistics.com/tests/studentttest/default.aspx>

Another approach to determining the relationship between two sample means is to test the differences between the matched or paired variables. In this case, you would be attempting to determine if there is a greater than zero difference between the samples. The hypotheses would be established as follows:

$$H_0: \mu_d = 0$$

$$H_a: \mu_d > 0$$

The Level of Significance

When a researcher conducts a study, the level of statistical significance is typically determined before the hypothesis test is performed. The level of significance, α , the probability of rejecting the null hypothesis when it is true. For example, a significance level of .05 indicates a 5% risk of rejecting a correct null hypothesis. While it is possible to select any value for the level of significance, researchers will select the lowest percentage that they feel will be acceptable for their research. The most common level of significance is $\alpha=.05$ which also implies a 95% confidence level.

The P-value

The results of a hypothesis test will always report a p-value. The p-value represents the probability associated with the test statistics assuming that the null hypothesis is valid. In practice, the p-value is used as a decision value when compared to the selected level of significance α to determine if the null hypothesis should be rejected. A small p-value will mean that there is strong evidence to support the alternative hypothesis.

If $p\text{-value} < \alpha$, then *reject the null hypothesis*. This means that there is sufficient evidence to support the alternative hypothesis.

If $p\text{-value} \geq \alpha$, then *fail to reject the null hypothesis*. This result means that there is not sufficient evidence to support the alternative hypothesis.